Research at High Pressure (Primary Pressure Standards)

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Abstract

Controlled-clearance piston gages are used as primary standards to establish and measure high pressures. The uncertainty associated with the clearance between the piston and cylinder in a controlled-clearance gage is a major contributor to the total uncertainty of most gages. The clearance is typically estimated by measuring the flow of the pressurizing fluid through the annulus as a function of pressure. This paper compares two models for estimating the clearance with recent data.

Introduction

The pressure, defined as the force per unit area, generated by a controlled-clearance piston gage is a function of the geometrical area of the piston, the forces applied to the piston and the conditions in the crevice between the piston and the cylinder. The dominant force is that generated by the applied mass load which is well known. Other forces due to the viscous drag of the fluid on the piston are harder to quantify. The viscous forces are typically estimated by indirectly measuring the flow of fluid through the crevice between the piston and cylinder as a function of the applied load and the pressure applied to the outside of the cylinder (jacket pressure used to control the clearance between the piston and cylinder). A simple schematic of a controlled-clearance gage is shown in Fig. 1. The distortion of the piston is determined from elasticity theory using the material properties of the piston. It is difficult to determine the area of the cylinder, and hence the clearance between the piston and cylinder, and how it changes with pressure. Conceptually, in a controlled-clearance gage, the cylinder can be partially collapsed around the piston with the application of a jacket pressure, P_j, of sufficient magnitude. The piston and cylinder would then have the same diameter and thus the area of a portion of the cylinder would have been determined since the piston can be accurately measured. However if this is done, the gage can be damaged. A compromise evolved such that the cylinder was not squeezed down beyond a point estimated to be safely away from the point at which damage would be done to either the piston or the cylinder but close enough so that the clearance between piston and cylinder would be reduced. As a result of this compromise it is still necessary to estimate the size of the crevice between the piston and cylinder by indirectly measuring the fluid flow thorough the crevice as a function of the jacket pressure and load pressure. Uncertainties of the models which determine the crevice width from the measured flow are major contributors to overall gage uncertainty.

In the following paragraphs we will first describe the connection between piston fall rates, and the crevice widths and how this contributes to the effective area of the gage. Physical models of the gage are often employed to make these connections. We will describe two separate models which we used for this purpose. The first originated with Dadson et al[1] and we have modified it slightly to allow for controlled-clearance jacket pressures. The second is referred to as the Heydemann-Welch model[2] which has been used to describe controlled-clearance gages. Both models can accommodate important aspects of the present data taken from a 2.5 mm diameter gage operating between 50 MPa and 700 MPa. But systematic deviations that cannot be accommodated by either model as well as the presence of unphysical implications remain. The end result is that uncertainties for the effective area of the gauge due to the model must be increased as much as 120 parts per million over and above the total uncertainty due to other sources.

Theory

Typically the crevice widths between piston and cylinder are estimated by measuring fall-rates of the piston, $\dot{z}_p = dz_p/dt$, which are proportional to the fluid flow. Here z_p is the vertical position of the piston and its time derivative is referred to as the fall rate. Fall-rate measurements can be related to crevice width through the Poiseuille flow equation[4,5]:

$$J = \frac{\pi R\, n\, h^3(z,p)}{6\,\eta\,(p)}\frac{dp}{dz} \quad , \tag{1}$$

where J is the molar flow through the crevice, R is the piston radius, n is the molar density of the fluid, $h(z,p)$ is the radial crevice width (in general a function of both z, the local coordinate, and p, the local pressure inside the crevice), $\eta(p)$ is the fluid's dynamic viscosity as a function of pressure and dp/dz is the derivative of the pressure in the crevice with respect to the vertical coordinate. J is related to $\dot{z}_p$ through:

$$J = n\,\pi R^2\ \dot{z}_p \tag{2}$$

Dadson et al[1] showed, that a measurement of the piston fall-rate is essentially a measurement of the crevice width cubed and averaged in a certain way over the engagement length. For example if $h(z)$ is assumed to be a function only of z then one can obtain the following equation by equating Eqs. 1 and 2 and integrating over z and p;

$$\dot{z}_p = \frac{1}{6\,R}\left(\frac{\int_0^P \frac{dp}{\eta(p)}}{\int_0^L \frac{dz}{h^3(z)}}\right) \quad ; \tag{3}$$

or if $h(p)$ is assumed to be a function only of p as in the model developed by Dadson et al then;

$$\dot{z}_p = \frac{1}{6\,R\,L}\int_0^P \frac{h^3(p)}{\eta\,(p)}dp \tag{4}$$

It is difficult to model this analytically if $h(z,p)$ is a function of both z and p. Although it is realized that $h(z,p)$ is a function of both variables, the dependence on p for a properly manufactured gage is predominant. The effect of the viscous force is combined with the geometrical area to form an effective area defined as:

$$A_{eff} = A_0\,(1 + bP) + \frac{F_v}{P} \quad , \tag{5}$$

where A_0 is the geometrical piston area, b is the distortion coefficient of the piston, P is the pressure beneath the piston and F_v is the viscous force. Dadson et al have shown that the force the viscous fluid exerts on the piston is given by the following equation:

$$F_v \approx \frac{A_0}{R}\int_0^P h(p)dp \tag{6}$$

Notice that F_v is an average of the crevice width. Substituting Eq. 6 into Eq. 5 yields an expression for the effective area in terms of an integral over the crevice width.

$$A_{eff} = A_0 \left(1 + bP + \frac{1}{RP} \int_0^P h(p)dp \right) \tag{7}$$

As seen here, to calculate effective area[1] one requires an average crevice width rather than the quantity that is typically measured in a fall-rate measurement (the average of crevice width cubed, Eq. 4).

If pistons and cylinders were perfectly straight and infinitely resistant to deformation then $< h^3 >$ and $< h >^3$ would be interchangeable and a measurement of the fall-rate would determine the effective area. Or if a good fit to the experimental fall-rate data were obtained using Eq. 4 and a model for $h(p)$, then one would have confidence in the model and one could then proceed to estimate the effective area due to viscous forces alone using Eq. 7. However, it is often the case that residuals of the fall-rate data from the fits show significant systematic differences. If this is the case, as it was in the present investigation, then one cannot reliably convert the fall-rate data to area information via Eqs. 4 and 7 even though fall-rate data themselves are often of sufficient accuracy and precision. Past practice has been to assume the essential correctness of a given model for the crevice width and to proceed with the analysis.

Other information regarding crevice widths can be obtained from measuring the derivative of the effective area with respect to the jacket pressure, referred to as "d" measurements:

$$d \equiv -\frac{1}{A_{eff}} \frac{dA_{eff}}{dP_j} \approx -\frac{1}{RP} \frac{d}{dP_j} \int_0^P h(p)dp \quad . \tag{8}$$

We have examined two models by fitting them to fall-rate and "d" measurements taken with a 2.5 mm diameter controlled-clearance piston gage. The first model is due to Dadson et al[1] (modified for use on a controlled-clearance gage) and the second is a model based on Heydemann and Welch[2]. The fall-rate measurements are in the form of data triplets ($\dot{z}_p,\ P,\ P_j$) which can be thought of as a function with one dependent and two independent variables. This data can be fitted by analytical functions and we have used both a modified Dadson model and the Heydemann-Welch model for the present data. The Dadson model was originally developed for simple piston gages ($P_j = 0$) but has been modified in the present case to include controlled-clearance piston gages ($P_j \geq 0$).

Two Models

a) The modified Dadson model assumes a crevice width that is proportional to the local pressure in the crevice. In this model the crevice width is taken as:

$$h(p) = h_0 + \nu P + \mu p + \nu_j P_j \qquad , \tag{9}$$

where h_0 is the crevice width at $P = 0$, and at $P_j = 0$. h depends on the pressures P, P_j and the local pressure inside the crevice, p, which varies between P at the bottom of the piston and 0 at the top. The modified model contains four free parameters and is similar to a model discussed by Dadson et al[1] containing the three free parameters h_0, ν, and μ. The present model adds the parameter ν_j, (a linear jacket pressure term). The magnitude of the three parameters ν, μ, and ν_j can also be estimated from elasticity theory. In the Dadson model, $h(p)$ from Eq. 9 is substituted into Eq. 4. The viscosity can be represented by a polynomial form $\eta(p) = \eta_0(1 + cp)^q$ used by Vergne[5]. With these substitutions the Dadson model can be integrated analytically.

Figures 2 and 3 show a simultaneous fit of the fall-rate measurements and the "d" measurements by the model described by Eq. 9. Although the fit shown in Figs. 2-4 appears reasonable, contradictions appear when the fitting parameters are substituted into Eq. 4. For example, for some values of P_j for which the fall rates were greater than zero, ($\dot{z}_p > 0$) the crevice appears to close down and even become negative. After extensive investigation we have concluded that the data are of sufficient accuracy and it is the model that breaks down under these circumstances. In any case the resulting model predictions with regard to effective area cannot be trusted completely.

b) The Heydemann-Welch model parameterizes the effective area of the controlled-clearance gage as follows:

$$A_{eff} = A_0\left[1 + (\alpha_c + \alpha_p)(T - T_{ref})\right](1 + bP_n)\left[1 + d\,(P_z - P_j)\right] \tag{10}$$

where A_0 is the geometrical area of the piston at the reference temperature, T_{ref} and at atmospheric pressure, α_c and α_p are the linear thermal expansion coefficients for the cylinder and piston, T is the temperature of the piston and cylinder at the time of the pressure measurement, b is the pressure coefficient of the piston, P_n is the nominal pressure beneath the piston, P_j is the jacket pressure, d is the derivative of the area with respect to P_j defined above, P_z is the jacket pressure at which the clearance between the piston and cylinder is extrapolated to be zero.

The terms d and P_z in the Heydemann-Welch model are given by the following:

$$P_z = \left(P_{z0} + S_z\,W + Q_z\,W^2\right) \qquad , \tag{11}$$

and

$$d = \left(D + E\,W + F\,W^2\right) \qquad , \tag{12}$$

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where W ($\approx P_n A_0$) is the local weight of the applied mass and P_{z0}, S_z, Q_z, D, E, and F are fitting parameters. In their method, three of these parameters (P_{z0}, S_z, Q_z) are obtained from the fall-rate data by fitting the jacket pressures in terms of the other two variables $\dot{z}_p$ and W. Their method uses the equation:

$$P_j = \left(P_{z0} + S_z W + Q_z W^2\right) + \left(a_1 + a_2 W + a_3 W^2\right)(\dot{z}_p)^{1/3} \qquad , \tag{13}$$

which describes the principal features of the fall-rate data. Here a_1 , a_2 and a_3 are additional fitting parameters. All of the fall-rate data shown in Fig. 5 were fit simultaneously by Eq. 13. The other three parameters (D, E, F) were obtained from the "d" measurements fitted by Eq. 12. Figure 6 shows the "d" measurements together with the fit from equation 12. Figure 7 shows deviations of the fall-rate data from Eq. 13.

[Note: Because P_j is taken as the dependent variable in the Heydemann-Welch method, Eq. 13, the data in Fig. 5 were plotted with P_j and $\dot{z}_p$ as the ordinate and abscissa respectively. This is reversed from the Dadson model Fig. 2, in which Eqs. 4 and 9 are used.]

Conclusions

By measuring the fall-rates and hence the flow in the crevice for various P and P_j, and also "d" values, one determines the crevice parameters given in either Eq. 9 for the modified Dadson model or Eqs. 11 and 12 for the Heydemann-Welch model. The Heydemann-Welch model shows significant systematic deviations from the fall-rate and "d" measurements and cannot be easily forced to conform to the fall-rate data. The modified Dadson model, while fitting the fall-rate data better, yields values for the fitting parameters that are not physical for much of the data (i.e. the crevice becomes negative). This may be in part a result of the over simplifying assumption that the crevice width is only a function of p and not z. These developments necessitate that model uncertainties for a controlled-clearance gage such as the one used in this study can contribute as much as 120 ppm and should be added to the uncertainties due to other sources. Numerical methods are presently being developed here at NIST to improve this situation. It is hoped that with modern numerical methods in which the crevice can be a function of both z and p, we will be able to model successfully fall-rate and "d" measurements and reduce present uncertainties.

References

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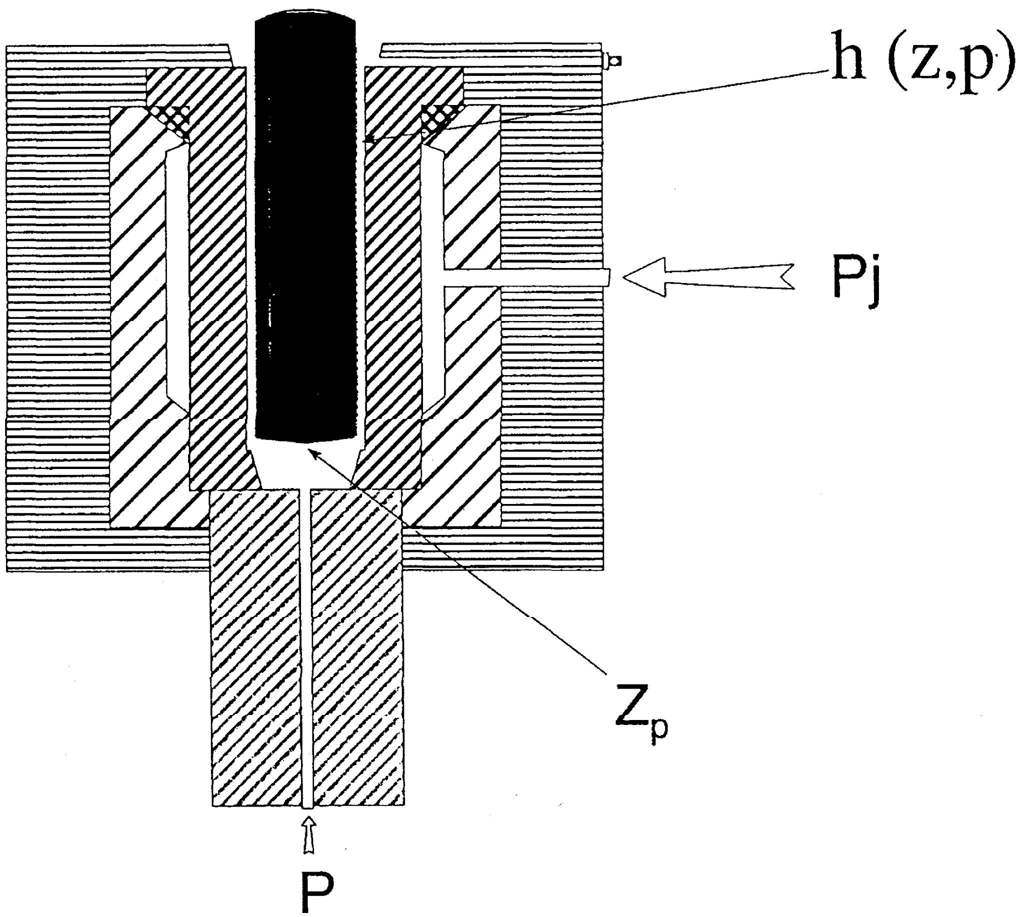

Figure 1. Schematic sketch of a controlled-clearance piston gage.

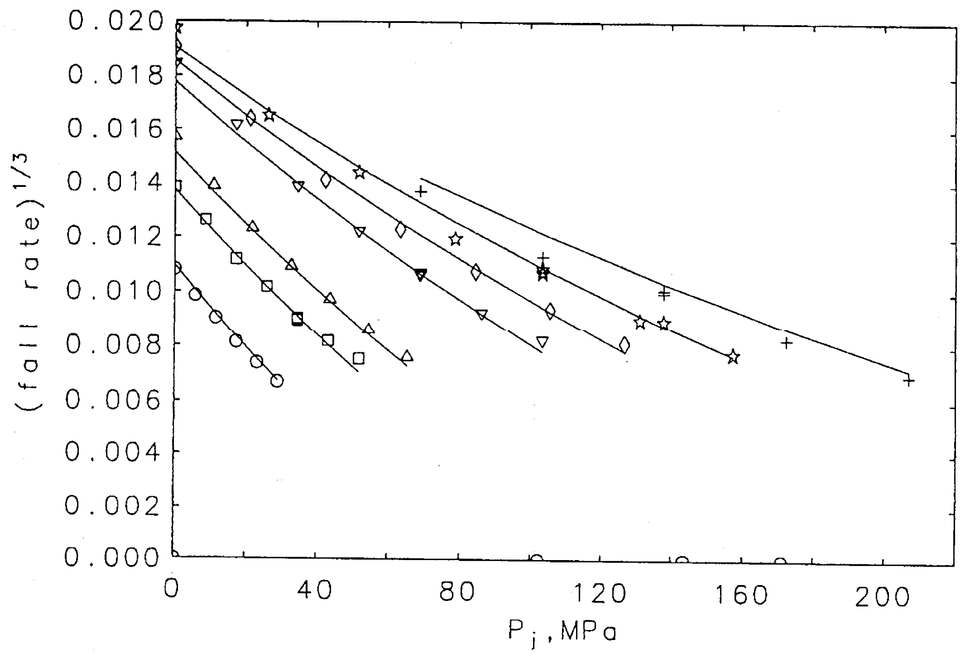

Figure 2. Piston (fall rate)$^{1/3}$ plotted as a function of jacket pressure, P_j. Symbols indicate pressures due to various loads, P; solid lines indicate a fit by the modified Dadson model.

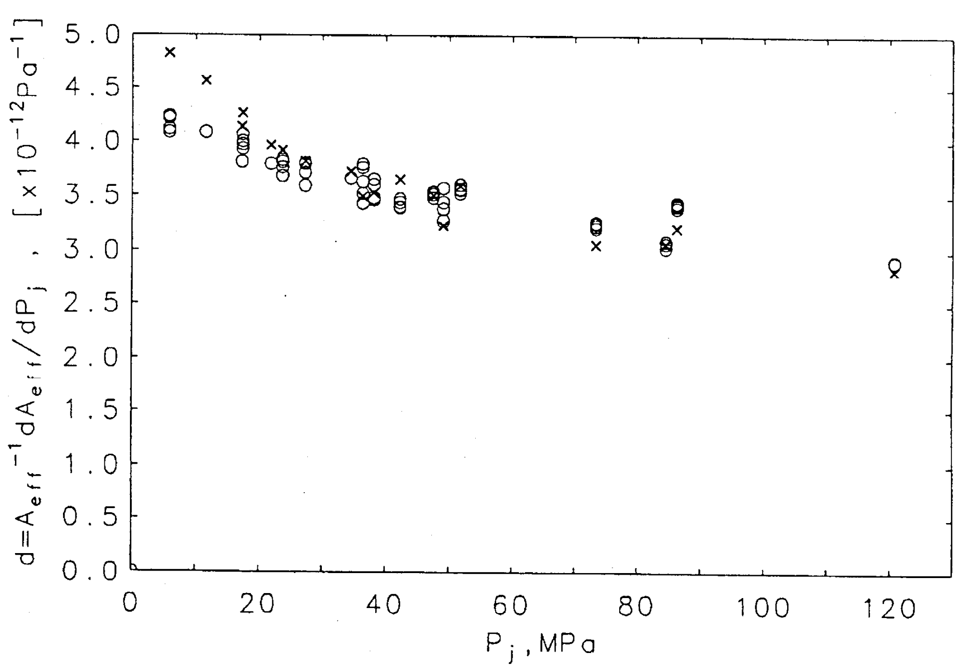

Figure 3. Derivative of effective area with respect to the jacket pressure, P_j, plotted as a function of P_j. Circles indicate measured data; x's indicate the fit by the modified Dadson model.

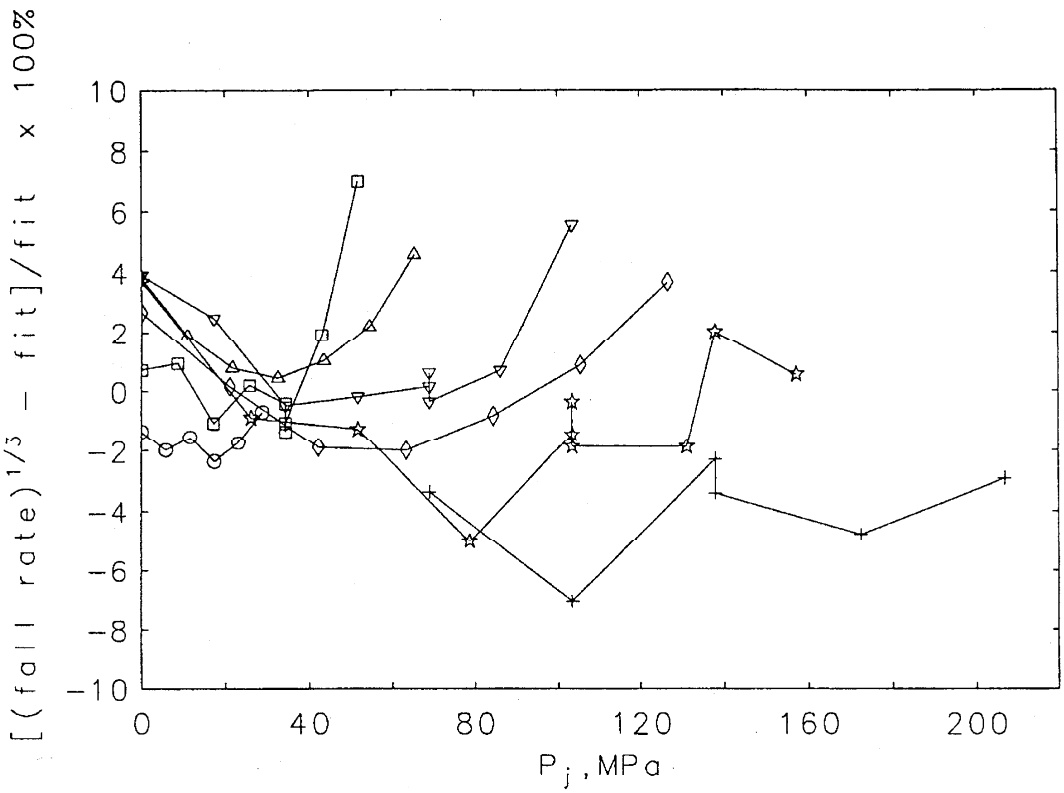

Figure 4. Deviations of data shown in Fig. 2 from the modified Dadson model plotted as a function of P_j.

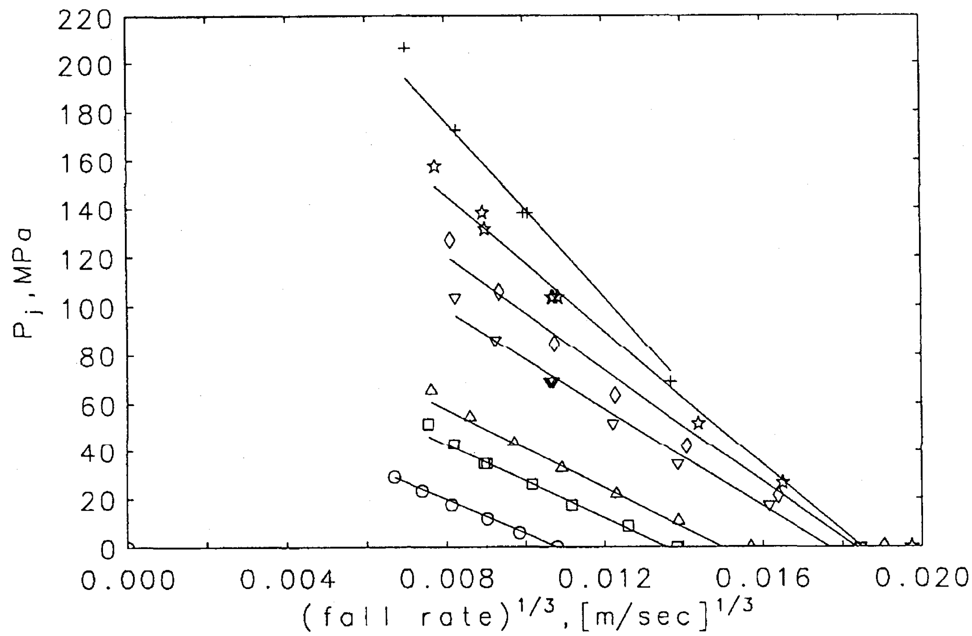

Figure 5. Jacket Pressure, P_j, plotted as a function of the cube root of the piston's fall-rate. *Symbols indicate pressures due to various loads, P up to 700 MPa; solid lines indicate a fit by the Heydemann-Welch model.*

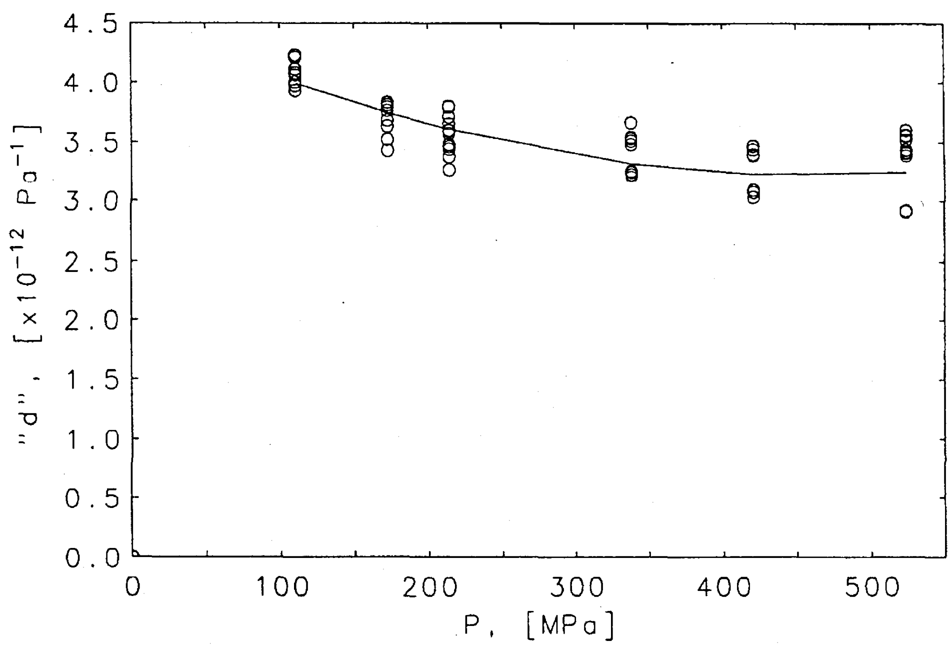

Figure 6. Derivative of effective area with respect to the jacket pressure, P_j, plotted as a function of pressure due to load, P. The solid line indicates a fit by the Heydemann-Welch model.

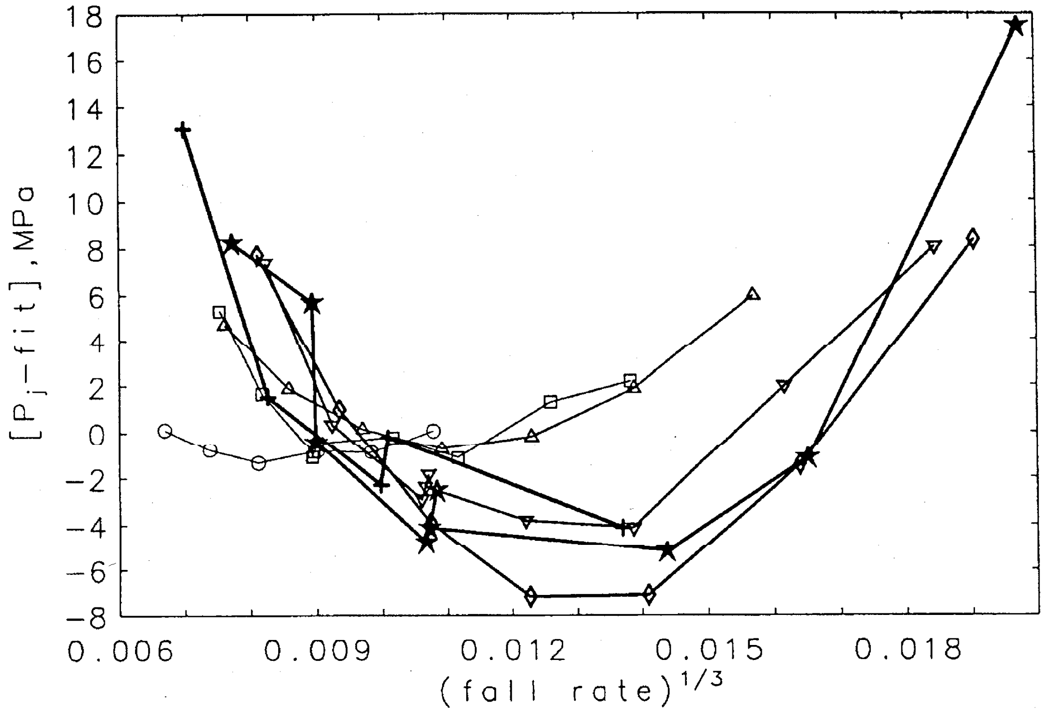

Figure 7. Deviations of data shown in Fig. 5 from the Heydemann-Welch model plotted as a function of the cube root of the piston's fall-rate.